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The Quantum Field Theory of Physics and of Mathematics[1]

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One recognizes that there has been, and continues to be, a great deal of common ground between statistical mechanics and quantum field theory (QFT). Many of the effects and methods of statistical physics find parallels in QFT, particularly in the application of the latter to particle physics. One encounters spontaneous symmetry breaking, renormalization group, solitons, effective field theories, fractional charge, and many other shared phenomena.

Professor Fisher [1] has given us a wonderful overview of the discovery and role of the renormalization group (RG) in statistical physics. He also touched on some of the similarities and differences in the foundations of the RG in condensed matter and high energy physics, which was amplified in the discussion. In the latter subject, in addition to the formulation requiring cutoff-independence, we have the very fruitful Callan–Symanzik equations. That is, in the process of renormalizing the divergences of QFT, arbitrary, finite mass-scales appear in the renormalized amplitudes. The Callan–Symanzik equations are the consequence of the requirement that the renormalized amplitudes in fact be independent of these arbitrary masses. This point of view is particularly useful in particle physics, although it does make its appearance in condensed matter physics as well.

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The very beautiful subject of conformal field theory spans all three topics we are considering: critical phenomena, quantum field theory, and mathematics. The relationship between conformal field theory and two-dimensional critical phenomena has become particularly fruitful in recent years. Conformal field theory in its own right, as well as being an aspect of string theory, has also played an important role in the recent vigorous interchanges between physics and mathematics. A wide variety of mathematical techniques have become commonplace tools to physicists working in these areas. Similarly, questions raised by conformal field theorists, topological field theorists, and string theorists have presented many new directions and opportunities for mathematics, particularly in geometry and topology. The relationship has been rich and symbiotic.

By contrast, it is my view that the more traditional mathematical physics of constructive and axiomatic field theory, and related endeavors, have not come any closer to the physicists' view of QFT. In fact, with the increased appreciation and understanding of effective field theories, the gap between the two communities might even be widening, if I understand correctly. It is this issue that I wish to address.

It should be clear that the quantum field theory of mathematics is very different from that of physics. In fact, I have long had the opinion that these may not even be the same theories at all. That is, there are (at least) two classes of quantum field theories, which for historical reasons go by the same name, despite being very different. The quantum field theory considered by the mathematics community is built on an axiomatic structure, and requires that the infinite volume system be consistent at <u>all</u> distance scales, infrared as well as ultraviolet. By contrast what physicists mean by a field theory is, in contemporary language, an effective field theory, which is applicable within a well-defined domain of validity, usually below some energy scale. Consistency is not required at short-distances, *i.e.*, at energies above a specified energy scale.

Does any four dimensional QFT exist, in the mathematical sense? The status of this topic was reviewed by Jaffe [2]. As yet, no four-dimensional QFT has been demonstrated to meet all the necessary requirements, although it is believed that pure Yang–Mills theory will eventually attain the status of a consistent theory. What about the standard model? There are two possibilities. Either it is just a matter of time for the necessary ingredients to be assembled to produce a mathematically consistent four-dimensional QFT describing the standard model, or no such theory exists. Suppose a given candidate field theory of interest is shown in fact not to be consistent. One possible response is to embed the candidate theory in a larger system with more degrees of freedom, *i.e.*, additional fields, and reexamine the consistency of the enlarged system. The hope is that eventually this sequence stops, and there is a consistent QFT. However to repeat, the logical possibility exists that this procedure does not lead to a consistent four-dimensional local QFT.

If no such consistent local QFT exists this would not have grave consequences for physical theory as we know it. From the physicists' point of view, the nesting of a sequence of theories is a familiar strategy carrying a physical description to higher energies. Embedding QED in the standard model, and then into possible grand unified QFT, extends the description of fundamental interactions to at least 100 Gev, and hopefully to still higher energies. However, the last <u>field theory</u> in this sequence may not be consistent at the shortest distance scales in the mathematical sense. In any case, eventually in this regression to shorter-distances, quantum effects of gravity are encountered. The question of consistency then must change. Then it becomes plausible that to proceed to even shorter distances, something other than local QFT or <u>local</u> quantum gravity will be required. String theory provides one possibility for an extension to distances where quantum gravity is relevant. The standard model is then just an effective low-energy representation of string theory. It has even been speculated, on the basis of string strong-weak duality, that string theory itself may

only be an effective theory of some other theory (for example the as yet unknown M-theory) [3]. Therefore, the physicist does not (need to) care if there is any completely consistent local QFT, valid at the shortest distances. The appropriate theories to be considered are effective theories; usually, but not always effective field theories.

Jackiw [4] argues elegantly and persuasively for physical information carried by certain infinities of QFT; making their presence known in anomalies and spontaneous symmetry breaking. In QFT, the ultraviolet divergences should not just be regarded as awkward flaws, but rather a feature which can lead to physical consequences. However it should not be concluded from his analysis that a theory without ultraviolet infinities cannot describe the physical phenomena appropriate to anomalies and spontaneous symmetry breaking. Such phenomena can be accommodated without ultraviolet divergences; string theory again providing one example, although the language to describe the phenomena will differ. Though ultraviolet divergences are an essential feature of local QFT, or effective field theory, they are not necessary for a description of the physics. Jackiw asks whether the string theory program has illuminated any physical questions. I should like to respond briefly in the affirmative. String theory has provided us with finite, well defined examples of quantum gravity; the only class of such theories presently known, with an essential aspect being non-locality. It had long been conjectured that one could not construct a non-local theory which was Lorentz invariant, positive definite, causal and unitary. One wondered whether there was any finite quantum gravity. Certainly string theory sets these earlier prejudices aside and allows us to consider a much broader class of theories in confronting the physical world. One should acknowledge that these are issues that have long been on the physicists' agenda, and are not solely of mathematical interest. In this evolution of our understanding one still retains the basic assumptions of quantum mechanics, even though locality is no longer a sacred physical principle.

Mathematicians speak of field theories such as quantum electrodynamics (QED) as heuristic field theories, or even as models, since no proof of mathematical consistency, at all distances, exists in the sense mentioned earlier. I feel that this description of QED is pejorative, albeit unintended. In fact QED is the most precise physical theory ever constructed, with well-defined calculational rules for a very broad range of physical phenomena, and extraordinary experimental verification. There is even a plausible explanation of why the fine-structure constant is small, based on a renormalization group extrapolation in grand unified theories. Of course, we understand that QED is an effective field theory, but is a well-defined theory in the sense of physical science. We know that to extend its domain of validity one may embed it in the so-called standard model (itself an effective field theory), the electro-weak sector of which has been tested over an enormous energy range (ev to 100 Gev; 11 decades), although not with the precision of QED. Thus both QED and the standard model are full-fledged theories of physical phenomena in every sense of the word!

The investigation of the mathematical consistency of four-dimensional local QFT is an interesting question in its own right. No matter what the outcome, we will gain important insights into the structure of QFT. However, the answers to such questions are not likely to change the way we do particle physics. Then how can mathematical physics make contact with issues of concern to particle physics? What are the right questions? Some suggestions immediately come to mind. What is a mathematically consistent <u>effective</u> field theory? Is this even a well-posed problem? If so, what restrictions does it place on effective theories? Can any candidates be discarded? To begin with, one should not expect that effective field theories are local field theories, as they involve infinite polynomials in fields and their derivatives. Nor do they have to be consistent at the shortest distances. An approach to some of these issues has been made by Gomis and Weinberg [5] "Are Non-renormalizable Gauge Theories Renormalizable in the Modern Sense?" They require that the

infinities from loop graphs be constrained by the symmetries of the bare action, such that there is a counterterm available to absorb every infinity. This is a necessary requirement for a theory to make sense perturbatively. Their criteria are automatically satisfied if the bare action arises from global, linearly realized symmetries. However, this becomes a non-trivial requirement if either there are non-linearly realized symmetries or gauge symmetries in the bare action. In constructive field theory one encounters a cutoff version of the local field theory being studied at intermediate stages of the analysis. These can be regarded as effective field theories, but to be relevant they must be Lorentz invariant. However, one needs to consider a wider class of effective theories than presently considered by constructive field theorists if the work is to have an impact on the concerns of particle physicists. In any case, there is certainly a great deal more to do in making effective field theories more precise.

In summary, I have argued that the QFT of mathematicians and of physicists are quite different, although both go by the name of QFT. To bridge the gap, one recognizes that there are many important mathematical problems posed by effective field theories, but these have not received the attention they deserve. Further, the existence of consistent string theories challenges the idea that locality is essential in the description of particle physics at the shortest distances.

References

[1] Michael E. Fisher, "Renormalization Group Theory: Its Basis and Formulation in Statistical Physics," presented at this conference.

[2] Arthur Jaffe, "How Quantum Field Theory Fits into the Big Picture," presented at this conference.

[3] Edward Witten, Loeb Lectures, Harvard University, Fall 1995.

[4] Roman Jackiw, "The Unreasonable Effectiveness of Quantum Field Theory," MIT–CTP–2500, hep-th/9602122, presented at this conference.

[5] Joaquim Gomis and Steven Weinberg, "Are Nonrenormalizable Gauge Theories Renormalizable in the Modern Sense?" RIMS–1036, UTTG–18–95, hep-th/9510087.